August 27-31, 2023
FLORENCE - ITALY
16th International
Conference on
Wind Engineering

Generalized vortex induced vibration model and extension to tandem arrangement

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SUMMARY:
This paper proposes a general overview of current Vortex-Induced Vibration (VIV) models by showing their equivalence in a general form. A new generalized model (GVIV) is proposed following the proposed generalized vortex shedding model (Rigo et al., 2022) of a static cylinder. In this case, the fluid equation for the lift force is a nonlinear oscillator combining all third order terms with an exogenous noise that reproduces the observed varying envelope of the lift force. Model parameters, together with the two coupling terms in the GVIV, are identified from lift force and structural displacement statistics from wind tunnel data. An extension to cylinders in tandem arrangement is then proposed, by doubling the same form of the fluid equation (a perturbed generalized vortex shedding model). Coupling terms between cylinders are added to take into account their interaction. These terms are proportional to cylinder lift variables with a phase that depends on the spacing between cylinders. This methodology is applied to wind tunnel measurements and gives promising results. The model can be extended to VIV by adding two structural equations.

Keywords: Vortex Induced Vibration, Circular cylinder, Tandem arrangement, Wind tunnel experimental testing, Wake-oscillator, Parameter identification, Nonlinear oscillator.

1. INTRODUCTION

The modelling of VIV can be classified into two families : (1) single degree-of-freedom (sdof) and (2) wake-oscillator models. The first ones focus on the structural equation with a fluid loading on the second member of different types. The second ones use a system of coupled differential equations for two variables: the structure and the wake. The structural equation is a harmonic oscillator excited by the wake variable on the second member. This variable is a representation of the wake, either as the lift force (Facchinetti et al., 2004; Hartlen and Currie, 1970), an equivalent variable whose derivative is the lift force (Krenk and Nielsen, 1999) or the angle of the wake lamina (Tamura, 1981). This paper shows that these different wake-oscillator models can be written in an equivalent general form with similar terms. For all models, the fluid equation is a nonlinear oscillator experiencing phenomenological properties of VIV: a self-excited and self-induced phenomenon. Rayleigh or Van der Pol oscillators are used in current models thanks to their well-known properties allowing self-sustained oscillations and the presence of a limit cycle. Nevertheless, the shape of this limit cycle in the phase plane does not matches observations. A new generalized lift force model is thus proposed with a nonlinear oscillator that contains all third order terms (Rigo et al., 2022). Coefficients are identified on wind tunnel (WT) data to reproduce the measured limit cycle. Lift force experimental measurements on a static cylinder show a varying envelope, coming from wake turbulence effects (Rigo et al., 2022). The new proposed model includes also an exogenous noise with a von Karman-like spectrum to reproduce fluctuations from wake turbulence. For a cylinder free to oscillate (in VIV), the lift force presents also a varying envelope and the structural displacement is not perfectly mono-harmonic, especially at the start and end of the lock-in range. When only the maximal amplitude is usually computed in VIV, the displacement envelope and standard deviation can be useful when the loading is repetitive and the structure is subjected to fatigue. Such results can be predicted thanks to the generalized Vortex Induced Vibration model (GVIV) proposed in this paper. It is built using the generalized fluid (lift force) equation from (Rigo et al., 2022), coupled with a structural equation. Parameters are identified using statistics of both the lift force and the structural displacement. In the present paper, the generalized VIV model is adjusted using a WT dataset of a spring-mounted rigid circular cylinder in VIV and the model is shown to be able to accurately replicate experimental measurements.

The generalized lift force model is versatile and is first extended to static circular cylinders in tandem arrangement by doubling the fluid equation and adding coupling terms between both cylinders to reproduce the flow coupling. The flow pattern around tandem cylinders is known to depend strongly on the spacing L/D between cylinders (center-to-center) and Reynolds number Re. In subcritical regime ($Re < 2 \cdot 10^5$), the flow pattern is divided into three types of wake interference (Zdravkovich, 1987): (i) extended-body regime where the cylinders behave like a single body ($1 < L/D < 1.2 - 1.8$), (ii) re-attachment regime where the shear layers re-attach from the front to the rear cylinder ($1.2 - 1.8 < L/D < L_c/D$), with $L_c/D \approx 4$ being the critical spacing and (iii) co-shedding regime where vortex shedding occurs behind both cylinders ($L/D > L_c/D$).

2. METHODS

The general features of wake-oscillator models are the same for all models: to generate a self-induced and self-limited motion thanks to generic nonlinear oscillators. Indeed, it is possible to obtain a general dimensionless expression for all wake-oscillator models. The dimensionless structural displacement is noted $Y = y/D$, where D is the cylinder diameter (or cross-flow dimension). A general harmonic oscillator for the structural equation is chosen. To reproduce the VIV features, a nonlinear oscillator is chosen for the fluid equation, with Q the fluid variable using a general nonlinear (third order) function $F(Q, Q')$. Coupling terms between fluid and structural degrees-of-freedom are added as general polynomial functions $G(Q, Q')$ and $H(Y, Y', Y'')$. Assuming $()'$ is the derivative with respect to the dimensionless time $\tau = \omega_0 t = 2\pi f_0 t$ and $\Omega = f_{vs}/f_0 = U_\infty/U_{cr} = StU_\infty/f_0 D$ (St is the Strouhal number, f_0 the natural frequency and f_{vs} the vortex shedding frequency), the general picture of wake-oscillators can be expressed as,

$$Y'' + 2\xi_t Y' + Y = G(Q, Q') \tag{1}$$

$$Q'' + \Omega F(Q, Q') + \Omega^2 Q = H(Y, Y', Y'') \tag{2}$$

Assuming that G and H are linear combinations of Q or Y (and their derivatives), the complete model is, with the total damping ξ_t including the structural and fluid ones,

$$Y'' + 2(\xi_s + \xi_a \Omega) Y' + Y = G_1 \Omega^2 Q + G_2 \Omega Q' \tag{3}$$

$$Q'' + \Omega \left(F_1 + F_2 Q^2 + \frac{F_3}{\Omega^2} Q'^2 \right) Q' + \Omega^2 Q = H_1 \Omega Y' + H_2 Y'' \tag{4}$$

Even with different model expressions, coefficients of all wake-oscillator models are very similar when expressed in their equivalent general form. The present model is obtained by adjusting the set of parameters $\pi = (\alpha, \gamma, \delta, \sigma_\eta, L_\eta, A_1, A_2, \xi_a)$ to minimize the objective function based on statistics of the lift force and structural displacement,

$$F(\pi) = \sum_i w_1(P(Q_i,\pi) - P^*_{Q_i})^2 + w_2(P(Q_{e_i},\pi) - P^*_{Q_{e,i}})^2 + w_3(P(Y_i,\pi) - P^*_{Y_i})^2 + w_4(P(Y_{e_i},\pi) - P^*_{Y_{e,i}})^2 \tag{5}$$

where $P^*_{Q_i}$ is the Probability Density Function (PDF) of the experimental lift force, $P(Q_{e_i},\pi)$ the PDF of the lift envelope from the model at parameters set π ($P^*_{Y_i}$ stands for the structure).

Table 1. Coefficients of wake-oscillator models and equivalence

	HC	Facchinetti	Tamura	Krenk	Present
$Q(\tau)$	$\frac{2C_L(\tau)}{C_{L0}}$	$\frac{2C_L(\tau)}{C_{L0}}$	$\frac{2f\alpha(\tau)}{C_{L0}}$	$\frac{2w(\tau)}{w_0}$	$\frac{2C_L(\tau)}{C_{L0}}$
ξ_a	0	$\frac{m_r\gamma}{2} = \frac{m_r C_D^*}{4\pi St}$	$\frac{m_r(f+C_D)}{4\pi St}$	0	ξ_a
F_1	$-\varepsilon_{HC}$	$-\varepsilon_F$	$-\varepsilon_T$	$-\varepsilon_K$	δ
F_2	0	ε_F	ε_T	ε_K	α
F_3	ε_{HC}	0	0	ε_K	γ
G_1	$\frac{C_{L0}}{2}a = \frac{m_r C_{L0}}{(4\pi St)^2}$	$\frac{m_r C_{L0}}{(4\pi St)^2}$	$-\frac{m_r C_{L0}}{(4\pi St)^2}$	0	A_1
G_2	0	0	0	$\frac{m_r C_{L0}}{2(2\pi St)^2}$	0
H_1	$\frac{2b}{C_{L0}}$	0	$-\frac{\pi St C_{L0}}{f}$	$-\frac{C_{L0}}{(v_0 4\pi St)^2}$	0
H_2	0	A	$-\frac{\lambda C_{L0}}{2f}$	0	A_2

Assuming $()'$ is the derivative with respect to the dimensionless time $\tau = \omega_{vs} t$ where $\omega_{vs} = 2\pi St U_\infty / D$, the proposed lift force model for tandem cylinders is obtained by doubling the present generalized fluid model and adding coupling terms between both cylinders,

$$Q_1'' - (\alpha_1 Q_1^2 + \gamma_1 Q_1'^2 + \delta_1)Q_1' + Q_1 = F_{21}(Q_2) + \eta_1 \tag{6}$$

$$Q_2'' - (\alpha_2 Q_2^2 + \gamma_2 Q_2'^2 + \delta_2)Q_2' + Q_2 = F_{12}(Q_1) + \eta_2. \tag{7}$$

In the re-attachment subcritical regime, the Strouhal number of both cylinders is about $St = 0.14$. Coupling terms are assumed to be proportional to delayed fluid variables (corresponding to the phase lag τ^* between cylinders), $F_{12}(Q_1) = F_{12}Q_1(\tau - \tau^*)$ (Facchinetti et al., 2002). Similarly to GVIV, coefficients are identified using an objective function based on cylinders experimental lift statistics.

3. RESULTS AND CONCLUSIONS

The WT dataset 1 was measured at the WTL of ULiège on a spring-mounted circular cylinder in subcritical regime ($Re = 10^4 - 4.4 \cdot 10^4$) with $D = 0.1$ m, $f_0 = 7$ Hz and $\xi = 0.1\%$. On Fig. 1, the generalized VIV model results at $\Omega = 1.07$ are compared with WT data. The model is able to reproduce correctly the fluid and structural dofs statistics thanks to the adjustment of nonlinear, additive noise and coupling terms $\pi = (-0.01, 0.09, 0.06, 0.87, 1.14, 0.002, 12, 0.004)$. Moreover, the parameters can be used in a prediction phase because their values are very close to those for the static cylinder.

The WT dataset 2 was measured on static tandem cylinders at ULiège (spacings $1.2 < L/D < 1.8$ and $Re = 4.5 \cdot 10^4$) (Dubois and Andrianne, 2022). Fig. 2 shows lift statistics results of the tandem

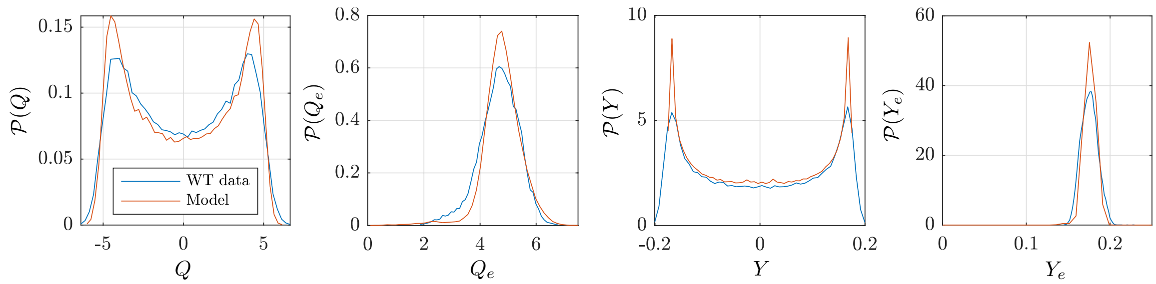

Figure 1. Results of the GVIV model identification: PDFs of Q, Q_e, Y, Y_e (WT dataset 1 at $\Omega = 1.07$)

lift force model identification for $L/D = 1.8$ and the model is able to reproduce the experimental ones. For this re-attachment regime, coefficients depend on the lift amplification Q_2/Q_1 and phase lag τ^*, strongly dependent on the spacing. This work opens several perspectives. Among others, it offers a simple and robust way to identify nonlinear coefficients in the GVIV model. Tandem lift force model can be investigated for other spacing types and extended to VIV of tandem cylinders.

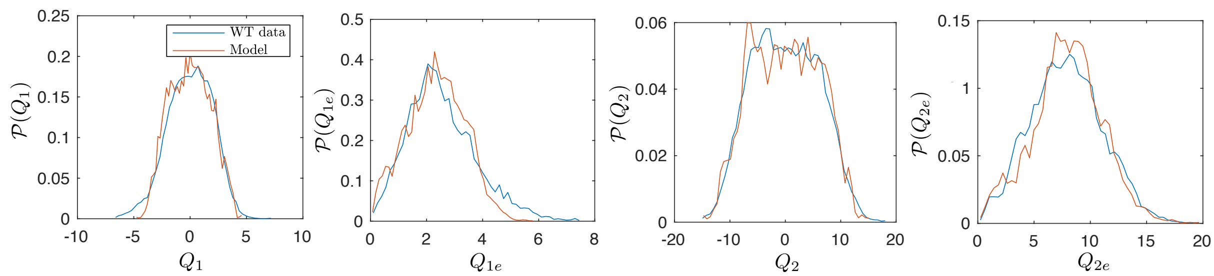

Figure 2. Tandem lift force model identification: PDFs of Q_1, Q_{1e}, Q_2, Q_{2e}, (WT dataset 2 for $L/D = 1.8$)

ACKNOWLEDGEMENTS

This study was made possible thanks to the support of the National Fund for Scientific Research of Belgium (FNRS).

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